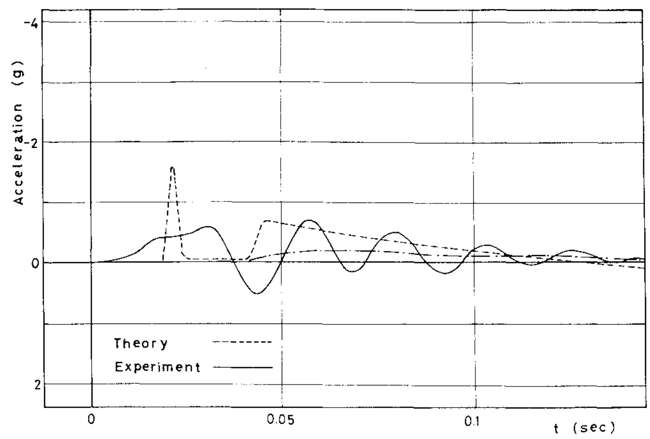

Fig. 4 Vertical acceleration of c.g. $h = 6$ in.

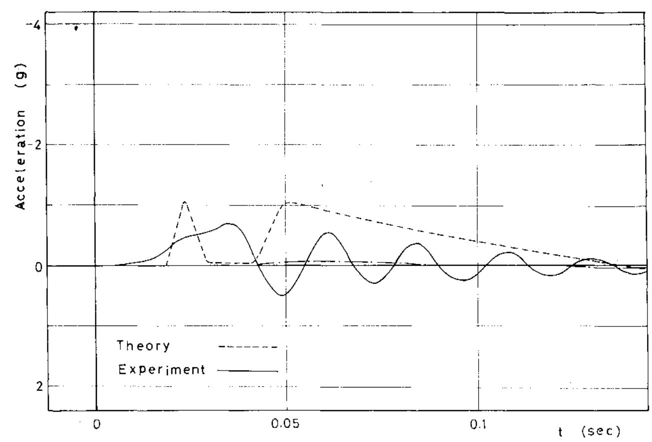

Fig. 5 Vertical acceleration of c.g. $h = 10$ in.

Experimental Results and Comparison with Theoretical Values

The variation of the normal acceleration at the c.g. can be calculated theoretically when the airplane encounters a gust.[5] It is assumed that the airplane is rigid and that the wavelength of the gust is sufficiently long compared with the full length of the airplane. The gust velocity profile is approximated as a triangular form. Therefore, the induced angle of attack α_g can be expressed by supperposition of three ramp inputs. Laplace transformed form is given by

$$\bar{\alpha}_g = \frac{\alpha(1-2e^{-T_l s}+e^{-2T_l s})}{s^2} \tag{1}$$

where a is the inclination of the gust velocity profile and T_l is the nondimensional time duration for the airplane to pass through the halfwidth of the gust. The external forces are, therefore, given by the following equations:

$$\Delta\bar{F}_t = C_{z\alpha}\bar{\alpha}_g \tag{2}$$

$$\Delta\bar{F}_m = C_{z\alpha}(S_t/Sw)\hat{l}_t e^{-Ts}\bar{\alpha}_g \tag{3}$$

Equation (2) expresses the aerodynamical force on the wing, and Eq. (3) expresses the pitching moment due to the force on the tail. Here, only the short-period mode is considered. Substituting the above equations into the longitudinal equations, the acceleration of the airplane c.g. can be calculated by inverse Laplace transformation.

Experimental results and theoretical values are compared in Figs. 4 and 5. These figures show that the experimental results differ considerably from the theoretical values. First, an oscillation of very short period (0.02 s) appears in the experimental results. This oscillation is caused by the elastic oscillation of the wing. Next, although the impulsive response and the short-period mode appear separately in the theory, both phenomena overlap with each other in the experimental results. This may be considered as the effect of the wide fuselage and the sweptback wing of the airplane. The dashed line in these figures shows the theoretical short-period mode of the model, which agrees favorably with the experiment. However, the level of acceleration in the experiment is lower than that calculated. This tendency becomes stronger in Fig. 5. Similar results were obtained for the pitching angular velocity.

Conclusion

In order to examine the gust response of a mid-sized jet transport, free-flight experiments are performed using a dynamically similar model. The vertical acceleration is acquired through telemetry. The results do not agree with theoretical values and the effect of the elastic distortion of the wing on the response exceeded our expectation. These results should be compared with theoretical values by the panel method which takes the elasticity into consideration.

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Some Nonlinear Effects in Stability and Control of Wing-in-Ground Effect Vehicles

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Nomenclature†

h	= height of mean quarter-chord point above ground
i_y	= radius of gyration of the aircraft in pitch
s	= Laplace transform variable
s_n	= unit of length = $v_n\ t_n$
t_n	= unit of time = v_n/g
$\hat{v}$	= nondimensional speed = V/v_n
v_n	= unit of speed = $\sqrt{\dfrac{W/S}{\rho/2}}$

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†The notation is based, where not specially explained, on the symbols of B. Etkin, *Dynamics of Flight,* John Wiley, New York, 1959.

$\hat{z}$ = nondimensional height of c.g. above ground $= z/s_n$
μ = relative mass parameter $= 2W/\rho Sg\bar{c} = s_n/\bar{c}$
τ = nondimensional time $= t/t_n$

Subscripts

0 = denotes initial state
h = denotes derivatives due to $h/\bar{c}$

I. Introduction

WHEN a three-dimensional wing is flown close to the ground, it experiences a decrease in induced drag and an increase in lift curve slope together with a nose-down pitching moment.

The performance and economic advantages of vehicles flying just above the surface has stimulated the concept of "wing-in-ground effect vehicles" (WIG's) intentionally operating within ground effect.[1-3] The benefits of increased lift/drag ratios are only gained when the WIG's are operated at heights of less than about 20% of the span. By fitting endplates to the tips of a low-aspect-ratio ram-wing, an increased lift/drag ratio is obtained with higher wing ground clearance.[4-7] An interesting potential for generating wing lift at low speed is the "power augmented ram-wing" (PAR) concept investigated by Gallington and Krause.[8] PAR might prove to be of great practical interest in water-based WIG design by providing means for avoiding high hydrodynamic drag during takeoff and high impact loads during takeoff and landing.

In comparison to the performance aspect, stability and control problems of WIG's have been investigated to a minor degree. Kumar[9] studied the stability of special configurations with simplifying assumptions, showing that the longitudinal modes were unstable and artificial stability was necessary. In Ref. 10 the static and dynamic stability of WIG's are treated more generally in a linearized analysis. A survey of the more important methods and results of Ref. 10 is given in this Note, extending the investigation into the area of nonlinear effects.

II. Analysis

At very low heights the aerodynamic coefficients in the longitudinal equations of motion are essentially nonlinear functions of height h and angle of attack α. Therefore the concept of aerodynamic derivatives cannot strictly be applied, because the derivatives can only be assumed constant for small perturbations. In this Note, quasilinear motion is used only for a more general description of static and dynamic stability, whereas important nonlinear effects are investigated without any linearization. This Note is limited to the very important longitudinal motion.

A. Equations of Motion

The longitudinal equations of motion are written in nondimensional units. Wind axes have been used as the reference axes. The three equations can be written as

$$\mathrm{d}\hat{v}/\mathrm{d}\tau = T/W - c_D \cdot \hat{v}^2 - \sin\gamma \tag{1}$$

$$\hat{v}\mathrm{d}\gamma/\mathrm{d}\tau = c_L \cdot \hat{v}^2 - \cos\gamma \tag{2}$$

$$\mathrm{d}^2\vartheta/\mathrm{d}\tau^2 = \mu(\bar{c}/i_y)^2 \cdot c_m \cdot \hat{v}^2 \tag{3}$$

We have also the kinematic relation

$$\mathrm{d}\hat{z}/\mathrm{d}\tau = \hat{v} \cdot \sin\gamma \tag{4}$$

Control inputs are included in the values of thrust and pitching moment. The nonlinearity of the aerodynamic coefficients can approximately be written in the form

$$c_D = f_D(h/b, \alpha) + c_{Dp} \tag{5a}$$

$$c_L = f_L(h/\bar{c}, \alpha) + c_{L_\eta} \cdot \delta\eta \tag{5b}$$

$$c_m = f_m(h/\bar{c}, \alpha) + \frac{1}{\mu\hat{v}_0}\left[c_{m\dot{\alpha}} f_{m\dot{\alpha}}\left(\frac{h}{\bar{c}}\right)\frac{d\alpha}{\mathrm{d}\tau} + c_{mq}\frac{\mathrm{d}\vartheta}{\mathrm{d}\tau}\right] \tag{5c}$$

where $f_{m\dot{\alpha}}(h/\bar{c})$ also accounts for nonlinearity. In the general case, Eqs. (1-5) have to be solved with given initial conditions and control inputs. Linearization of the equations is useful and acceptable if the effect of small disturbances on a rectilinear unaccelerated flight at constant height should be investigated. Then the essential influences of height and angle of attack on the aerodynamic coefficients Eqs. (5) can be written, using the concept of derivatives, in the form

$$\delta c_D = c_{D\alpha}\delta\alpha + \mu c_{Dh}\delta\hat{z} \tag{6a}$$

$$\delta c_L = c_{L\alpha}\delta\alpha + \mu c_{Lh}\delta\hat{z} \tag{6b}$$

$$\delta c_m = c_{m\alpha}\delta\alpha + \mu c_{mh}\delta\hat{z} + \frac{1}{\mu\hat{v}_0}[c_{m\dot{\alpha}}\delta\alpha' + c_{mq}\delta\vartheta'] \tag{6c}$$

where fixed controls are assumed.

Linearization of the equations of motion and inserting Eqs. (6) gives a characteristic equation of the fifth order,

$$As^5 + Bs^4 + Cs^3 + Ds^2 + Es + F = 0 \tag{7}$$

which normally has two oscillatory modes and an aperiodic mode.

B. Static Stability in Ground Effect

The criterion for static stability, in a general sense, is the condition $F>0$. As shown in Ref. 10, the coefficient F can be factored into three parts:

$$F \sim \left[-\frac{\partial c_m}{\partial\alpha}\right]_{\hat{v},h} \times \left[-\frac{\partial(T-D/W)}{\partial\hat{v}}\right]_{c_m, n=1} \times \left[-\frac{\partial n}{\partial(h/\bar{c})}\right]_{\hat{v},c_m} \tag{8}$$

The first factor is the "static pitching moment stability," well known as the "static stability" from analysis of flights under out-of-ground-effect conditions. The second term occurs in assessments of stability under constraints fixing the glide path angle and gives the "minimum drag speed" as the stability boundary. The third factor is a new specific term containing just the influence of the ground effect. It gives a necessary condition for a stable flight at constant height near the ground

$$\left[\frac{\partial n}{\partial h/\bar{c}}\right]_{c_m=0} \hat{v} = \text{const.} < 0 \tag{9}$$

and is called the "static height stability." The physical interpretation of Eq. (9) is as simple as the case for the "static pitching moment stability." Because the first two terms in Eq. (8) provide statically stable equilibrium both at pitching and speed disturbances, the condition of Eq. (9) gives a restoring force if the equilibrium in height is disturbed.

The static height stability can be expressed as a function of the aerodynamic derivatives by substituting Eqs. (6) in Eq. (9). This gives the expression

$$\left[\frac{\partial n}{\partial h/\bar{c}}\right]_{\hat{v},c_m} = \hat{v}_0^2 c_{Lh}\left(1 - \frac{c_{mh}c_{L\alpha}}{c_{m\alpha}c_{Lh}}\right) < 0 \tag{10}$$

In general, c_{Lh} is negative; thus the condition for static height stability can be expressed in the form

$$F_m \equiv \frac{c_{mh}c_{L\alpha}}{(-c_{m\alpha})(-c_{Lh})} < 1 \tag{11}$$

Table 1 Derivatives used in example (X-113)

$h/\bar{c}$	∞	1.0	0.4	0.168	0.084
C_{L0}	0.9	0.9	0.9	0.9	0.9
C_{D0}	0.12	0.10	0.08	0.06	0.04
$C_{L\alpha}$	3.0	3.2	3.6	5.6	7.9
C_{Lh}	0	−0.035	−0.35	−1.1	−2.9
$C_{D\alpha}$	0.86	0.82	0.70	0.82	0.72
C_{Dh}	0	0.001	0.002	0.005	0.008
$C_{m\alpha}$	−0.68	−0.70	−0.73	−0.85	−1.12
C_{mh}	0	0.003	0.055	0.100	0.210
$C_{m\dot{\alpha}}$	−1.33	−1.29	−1.20	−1.11	−0.98
C_{mq}	−4.44	−4.44	−4.44	−4.44	−4.44

All derivatives in Eq. (11) are rewritten as positive terms. If a wing approaches the ground, the lift coefficient increases, $(-c_{Lh})>0$, and in general a nose-down pitching moment results, $c_{mh}>0$. Therefore, to provide a WIG with a sufficient margin of static height stability, a high value of c_{mh} must be compensated by high values both of $(-c_{Lh})$ and of the static pitching stability $(-c_{m\alpha})$. The former term is mainly influenced by the airfoil characteristics of the wing. The latter derivative can be increased, without increasing c_{mh} as well, by a high horizontal tail working out of ground effect. Lippisch has used such a configuration very successfully with the X-113.[2] From Eq. (11), it is immediately seen that there is only a minor influence of c.g. position on static height stability due to the fact that c_{mh} and $(-c_{m\alpha})$ are changed in the same way.

C. Dynamic Stability in Ground Effect

The dynamic longitudinal stability of WIG's is determined by the roots of the fifth-order characteristic equation (7). Going out of ground effect, the coefficient F in Eq. (7) disappears in the same way that the static height stability, a factor of F, is reduced. This becomes the well-known quartic equation with the "short period mode" and the "phugoid mode" as the solutions. Flying near the ground, an aperiodic mode is obtained in addition to a pair of complex roots. The root-locus plot for a typical WIG configuration is given as a function of height in Fig. 1; derivatives used in this example are from Staufenbiel and Yeh[10] and are given in Table 1. The main conclusions drawn from this diagram are the following:

1) For the WIG configuration chosen in the example (X-113 type), dynamic stability is obtained in all modes if the vehicle flies near the ground ($h/\bar{c}\lesssim 0.5$).

2) The root-locus branch which moves away from the short-period roots, the "α-branch," shows an increase in the natural frequencies with a slight reduction of the damping ratios.

3) The "P-branch," emanating from the low-frequency low-damping phugoid mode, indicates an increase in both natural frequency and damping ratio below $h/\bar{c}\lesssim 0.5$. In a "transition phase" of heights ranging between $h/\bar{c}\approx 2$ and $h/\bar{c}\approx 0.5$, an unstable region is found. In this case artificial stabilization can be obtained by an airspeed-hold system.

4) A time constant, equal to the inverse of the negative value of the aperiodic root, describes the transients following speed disturbances. This time constant is reduced, as a favorable effect, in the "transition phase" but increases again in flights at very low heights. That is the influence of the reduction in induced drag.

D. Nonlinear Effects in Stability and Control

In ground effect the aerodynamic coefficients depend strongly on height and angle of attack, a fact which leads to nonlinear motions especially in climbs and descents as well as in responses to control inputs. The nonlinear behavior is

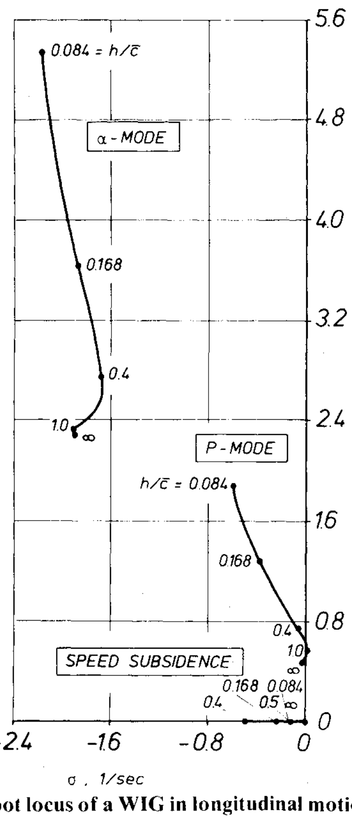

Fig. 1 Root locus of a WIG in longitudinal motion, $C_{L_0}=0.9$.

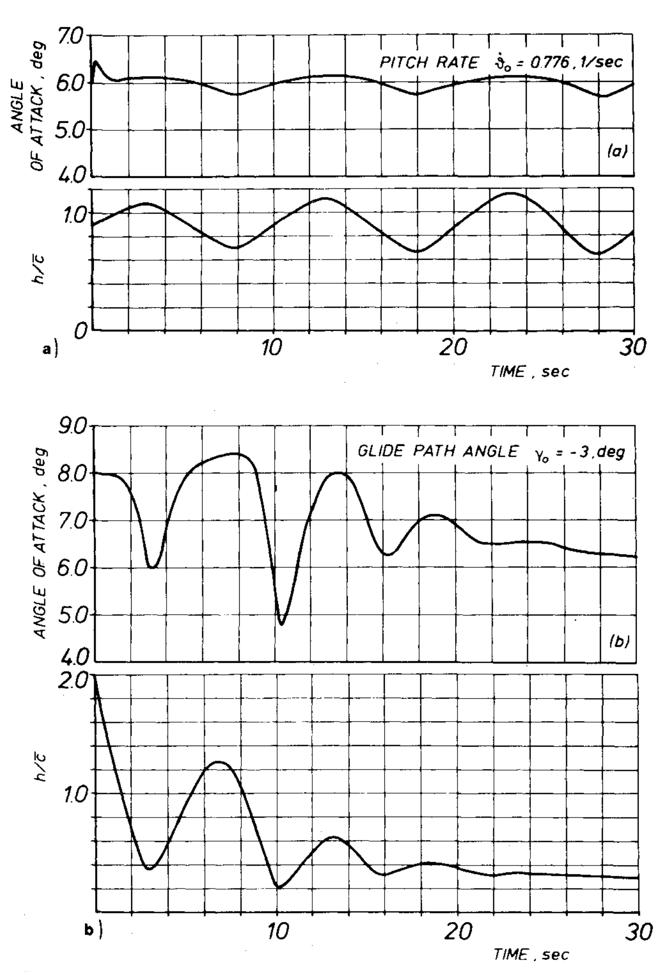

Fig. 2 Nonlinear characteristics of WIG's: a) limit cycle in "transition height regime"; b) "automatic" flare of a WIG. The nonlinear effects shown are based on simulations using aerodynamic coefficients derived from calculations and wind-tunnel tests which are not yet published.

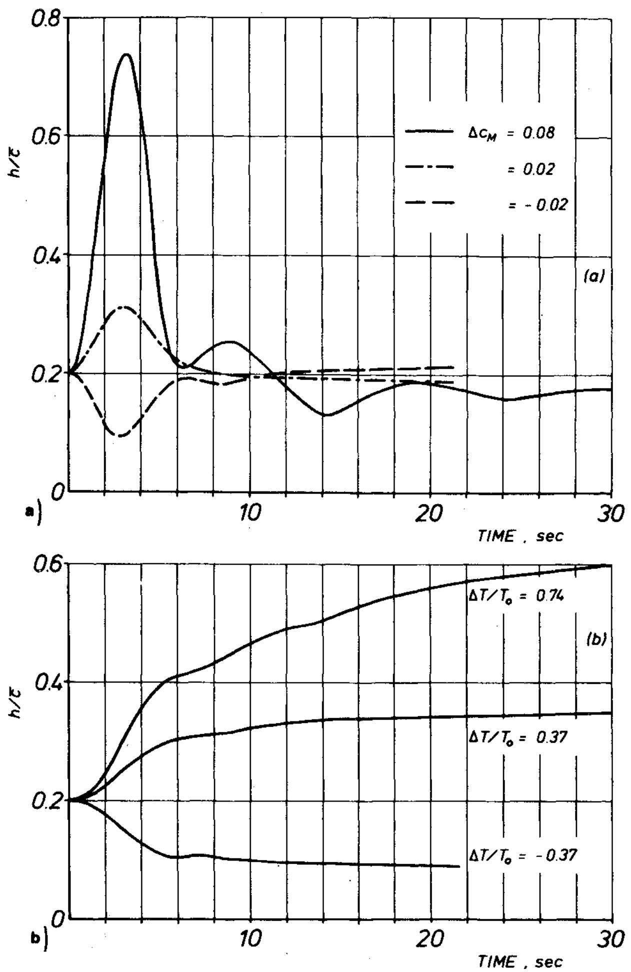

Fig. 3 Response to step input for a) pitching control and b) thrust control.

particularly observed when the flight path crosses the "transition height regime," $0.5 < h/\bar{c} < 2$. Within this transition regime a stable trimmed flight is not possible. At these heights the WIG will limit-cycle oscillate both in height and pitch motion (Fig. 2a).

Another very important nonlinear characteristic of WIG's is an "automatic flare" with fixed controls. In this case the increase of lift coefficient gives a spring-type force during approach to the ground, whereas the "vertical" kinetic energy is dissipated in several cycles of height and pitching motions. Figure 2b gives height and angle of attack in a three-degree approach with the "automatic" flare as a function of time. It is remarkable that the second minimum of height is the lowest one. Moreover, Fig. 2b shows a strong coupling between vertical motions and pitching motions, which are influenced by the static height stability, damping characteristics, and relative mass parameter. Several examples of vehicle response in ground effect due to control inputs are shown in Fig. 3. In principle, elevator and thrust inputs could be used for height and speed control. Application of the elevator mainly induces disturbances in both angle of attack and height, with only a small steady-state change in height (Fig. 3a). The danger of touching the ground or exciting heavy oscillations is a disadvantage of applying the elevator as a primary longitudinal control. In comparison to the elevator, the thrust control is a very favorable means of height control (Fig. 3b). Good steady-state changes in height can be obtained with small transients in angle of attack and height.

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